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CRACK GROWTH IN A SATURATED POROUS MEDIUM DUE TO PASSAGE OF A CURRENT PULSE

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A solution has been obtained [1] for the growth of a crack in a continuous medium in response to the thermoelastic stresses produced by passing a current perpendicular to the crack. Here we consider a model describing the action of a current pulse on a saturated porous medium when the current flows through a crack filled with liquid of high electrical conductivity. It is assumed that the medium has a skeleton of low electrical conductivity and is penetrated by capillaries filled with an electrically conducting liquid. Then the effective conductivity σ_0 is determined by the microcapillary conductivity. We consider the case where the direct current is passed through two coaxial elliptical cracks filled with liquid having a high conductivity σ_1. The crack opening is characterized by the parameter $\beta = c/l$, where c is crack width and l is length. If the crack is very much open ($\beta \gg \sigma_0/\sigma_1$), the current supplied to the center of the crack will emerge from the ends, and near the ends the current density will be maximal, as will the corresponding ohmic losses. The heating in the pores increases the pore pressure and may cause the medium to fail at the crack vertex. Here we use methods from the theory of complex variable functions to solve the two-dimensional problem on the current-density distribution around a crack, and the Biot theory [2] is used to discuss the consolidation of ground and to estimate the parameters of the electrical pulse that disrupts the medium.

1. Current-Density Distribution.

We consider the current-density distribution when the current flows through two elliptical cracks, with the source and antisource at the centers of these. We assume that the current is supplied through parallel infinitely long electrodes whose transverse dimension is much less than the crack width. The conductivity of the electrodes is much larger than that of the liquid within the cracks. This enables one to restrict consideration of the planar two-dimensional case. Figure 1 shows the track geometry. The potential distribution in a plane perpendicular to the electrodes satisfies the Laplace equation [3]

$$\operatorname{div}(\sigma_\nu \nabla \varphi_\nu) = -(I/2\pi)[\delta(z - z_1) - \delta(z - z_2)], \tag{1.1}$$

where $\nu = 1$ or 0. Here the potential φ_ν with subscript $\nu = 0$ corresponds to the region outside a crack, while that with subscript $\nu = 1$ corresponds to the region within the crack, $z = x + iy$ is the complex variable, x and y are Cartesian coordinates, z_1 and z_2 are the coordinates of the centers of the first and second ellipses correspondingly, I is the current injected into the crack per unit electrolength, and $\delta(z)$ is a Dirac function. The conditions for continuity of the potential and the normal component of the current density should be satisfied on the crack. We introduce the complex potential $F_\nu = \sigma_\nu \varphi_\nu$ and write these conditions in the form

$$\mathrm{Re} F_0 = \alpha \mathrm{Re} F_1; \tag{1.2}$$

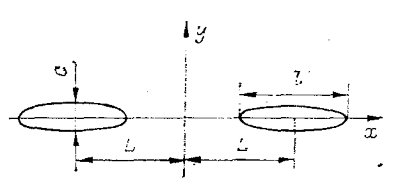

Fig. 1

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$$\mathrm{Im} F_0 = \mathrm{Im} F_1, \tag{1.3}$$

where $\alpha = \sigma_0/\sigma_1$. The expressions $\mathrm{Re}\, F_\nu = {}^1/_2(F_\nu + \bar{F}_\nu)$ and $\mathrm{Im}\, F_\nu = {}^1/_{2i}(F_\nu - \bar{F}_\nu)$ denote correspondingly the real and imaginary parts of the potential. Here and subsequently an overbar indicates that we take the conjugate value of the complex function. We add (1.2) and (1.3) to get a relation at the crack contour:

$$F_0 = \frac{\alpha+1}{2} F_1 + \frac{\alpha-1}{2} \bar{F}_1. \tag{1.4}$$

Also, the real parts of the potentials should be bounded in the regions of definition, apart from the points z_1 and z_2, at which lie the source and antisource. The potential F_0 may be determined in the form

$$F_0 = -\frac{I}{2\pi}\left\{\ln\frac{z-L}{z+L} + F_0^1\right\},$$

where F_0^1 is a function analytic in the region outside the crack. We perform a conformal mapping of the region outside the ellipses on the region outside unit circle. The mapping function takes the form

$$z \pm L = \omega(\xi) = R\left(\xi + \frac{n}{\xi}\right), \tag{1.5}$$

where $n = (l - c)/(l + c)$, $\xi = \rho\exp(i\theta)$, θ is the polar angle, and $2L$ is the distance between the centers of the cracks. In what follows, without loss of generality we assume that $R = 1$. The mapping of the crack contour onto the contour of unit circle occurs when $\xi = \sigma = \exp(i\theta)$. It has been shown [4] that F_0^1 can be represented as a Lorant series containing only terms with negative powers, as follows from the condition that the potential is bounded:

$$F_0^1 = -\frac{I}{2\pi}\left\{\sum_{k=1}^{\infty}\frac{a_k}{\xi^k} + \sum_{k=1}^{\infty}\frac{a_k}{[\xi(z-L)]^k} + \sum_{k=1}^{\infty}\frac{(-1)^{k+1}a_k}{[\xi(z+L)]^k}\right\}, \tag{1.6}$$

where a_k are the coefficients in the Lorant series. In writing (1.6), we have used the fact that F_0^1 is an odd function. Similarly, the potential F_1 can be put in the form

$$F_1 = -\frac{I}{2\pi}\left\{\ln\frac{z-L}{z+L} + F_1^1\right\},$$

where F_1^1 is a function analytic within an ellipse and can be represented as a series in Faber polynomials [4]:

$$F_1^1 = \sum_{k=1}^{\infty} b_k\left(\xi^k + \frac{n^k}{\xi^k}\right).$$

Then one can solve (1.1) by determining the unknown coefficients a_k and b_k, which can be found from (1.4). It follows from the symmetry that if F_0 and F_1 satisfy (1.4) on $z - L = \sigma + n/\sigma$, then they automatically satisfy that condition also on the contour $z + L$. Therefore, in what follows the entire discussion relates to the right-hand contour. The expression for the potential contains the small parameter $\varepsilon = 1/2L < 1/4$, which enables us to simplify the procedure for determining a_k and b_k substantially by using the small-parameter method. We use (1.5) and perform a series expansion in ε for the function $\xi(z + L)$, which is analytic on the contour of the right ellipse, to get up to terms in ε^4 that

$$\xi^{-1}(z+L) = \varepsilon - z^*\varepsilon^2 + \varepsilon^3(n + z^{*2}) - \varepsilon^4 z^*(3n + z^*)^2,$$

where $z^* = z - L$. We retain terms in the expansion with powers of ε not greater than 4, and then analogous expressions can be obtained for the function $\xi^{-k}(z + L)$. We substitute the expressions for F_1 and F_0 into (1.4) to get the function $\Phi(\sigma)$:

$$\Phi(\sigma) = F_0(\sigma) - \frac{\alpha+1}{2}F_1(\sigma) - \frac{\alpha-1}{2}\bar{F}_1(\sigma) = 0.$$

We specify that $\Phi(\sigma)$ is orthogonal to the system of functions $\sigma^{\pm k}$ $(k = 0, 1, 2, 3, 4)$, which gives us a system of algebraic equations for the a_k and b_k. The coefficients found by solving the system take the form

$$a_1 \approx \varepsilon\beta\left[1 + \frac{1+\varepsilon^2}{\alpha+0.5\beta}\right](1-\alpha), \quad b_1 \approx -\varepsilon\frac{1+\varepsilon^2}{\alpha+0.5\beta}(1-\alpha),$$

$$a_2 \approx -2\beta \frac{1+\varepsilon^2+2\varepsilon^4}{\alpha+2\beta}(1-\alpha), \quad b_2 \approx \frac{1}{2}\frac{1+\varepsilon^2+2\varepsilon^4}{\alpha+2\beta}(1-\alpha),$$

$$a_3 \approx \beta \frac{\varepsilon^3}{\alpha+1.5\beta}(1-\alpha), \quad b_3 \approx -\frac{\varepsilon^3}{3(\alpha+1.5\beta)}(1-\alpha).$$

The latter coefficients with accuracy up to ε^4 take the form

$$a_{2p+1} = b_{2p+1} = 0, \quad a_{2p} = (-1)^p \frac{n^p}{2p}\left[1 - \frac{(1+\alpha)\,n^{2p} + (\alpha-1)}{(\alpha-1)\,n^{2p} + (\alpha+1)}\right](1-\alpha),$$

$$b_{2p} = (-1)^p \frac{n^p}{2p}\frac{\alpha-1}{n^{2p}(\alpha-1)+(\alpha+1)}, \quad p=2, 3, \ldots, \infty.$$

For $\alpha \to 1$, the coefficients a_k and b_k tend to zero, and F_0 and F_1 give the solution for the potential distribution in an infinite plane containing a source and antisource, as would be expected, with these lying correspondingly at the points z_1 and z_2: $F_1 = F_0 = -(I/2\pi)\ln(z - z_1)/(z - z_2)$. It can be shown that for $\varepsilon \to 0$ and $\alpha \to 0$ we have $F_0 \to -(I/2\pi)\ln\xi$ and we get a known solution for a plane with a line of discontinuity, on which the value of the function is given as a constant. We note that as $\varepsilon < 1/4$, the interaction between the cracks has little effect on the solution, which virtually coincides with that for the case of a single crack ($\varepsilon = 0$). The current density j_x within the crack is given by $j_x = (\partial/\partial x)\mathrm{Re}\, F_1|_{y=0}$. As the main term in the expansion of F_1^1 is that containing b_2, we find the main term in the expression defining the current density near the vertex of the crack:

$$j_x(x_1) \approx \frac{I}{2\pi}\frac{1+\varepsilon^2+2\varepsilon^4}{\alpha+2\beta}x_1, \tag{1.7}$$

where x_1 is the reduced distance from the crack center. At the crack vertices, $|x_1| = 1 + n$. An important conclusion follows from (1.7). If the crack is very much open ($2\beta \gg \alpha$), then virtually all the current injected into the crack flows through the conducting liquid within it and emerges at the ends of the crack. The current density is dependent on c (crack width) and is defined by $j_x \approx I/\pi c$ near the vertex. If the crack is only slightly open, ($2\beta \ll \alpha$), only a small fraction of the current flows along it, since the resistance of the crack is large by comparison with that of the surrounding material. Then most of the current flows through the medium outside the crack and is hardly affected by the latter.

It is of interest to estimate the effective resistance of the medium per unit length of the electrodes, between which a potential difference ΔU is maintained: $R_{\mathrm{eff}} = \Delta U/I$. As $\sigma_1 \gg \sigma_0$, we get

$$R_{\mathrm{eff}} \approx \frac{1}{\pi\sigma_0}\left\{\ln\frac{8L}{l} - \frac{1+2\varepsilon^2+3\varepsilon^4}{1+\alpha/\beta}\right\}. \tag{1.8}$$

It is evident from (1.8) that the effective resistance is only slightly dependent on the distance between the cracks and is determined in the main by σ_0.

2. Damage to Medium. The heat production in the pore space near the vertex of a crack causes the liquid to expand and correspondingly raises the pore pressure, and if this rise is sufficiently rapid and the pressure cannot be dissipated by infiltration, then the porous medium may be damaged. We now estimate the parameters of the current pulse for which damage occurs. The following equations [5] describe the state of strain in the medium within the Biot ground consolidation theory:

$$\mu\Delta \mathbf{U} + \left(\lambda + \alpha_1^2 M\right)\nabla e = \alpha_1 M \nabla\zeta; \tag{2.1}$$

$$a\Delta\zeta = \partial\zeta/\partial t, \tag{2.2}$$

where u and λ are Lamé constants, U is the sleketon displacement vector, m is the porosity, e is the volume deformation of the skeleton, $m\varepsilon_1$ is the bulk deformation of the liquid, α_1 is the pore-pressure coefficient, mM is the compressibility modulus for the pore liquid, k_1 is the infiltration coefficient, $a = (\lambda + 2\mu)Mk_1/(\lambda + \alpha_1^2 M + 2\mu)$ is the consolidation coefficient, t is time, and $\zeta = -m(\varepsilon_1 - e)$.

Equations (2.1) and (2.2) describe the state of stress in the medium with allowance for the infiltration, which follows Darcy's law. Estimates show that the characteristic times for the infiltration processes are much less than the characteristic times for heat transfer between the liquid and the skeleton. Therefore, heat transfer is neglected. We consider the case where the energy is deposited in the pore space in a time $\Delta\tau_1 \ll c^2/\alpha$, i.e., virtually instantaneously. In that case, the pressure does not have time to be dissipated by infiltration, and the bulk deformation of the liquid is determined by ΔT, the temperature rise in the liquid in the pore space:

$$m\varepsilon_1 = -m\beta_1\Delta T, \text{ where } \Delta T = E/(c_l\,\rho_l). \tag{2.3}$$

Here β_1 is the thermal-expansion coefficient of the liquid, c_l and ρ_l are correspondingly the specific heat and density of the liquid, and $E = j^2\Delta\tau_1/(m\sigma_0)$ is the density of the energy deposited in the pore space. In writing the expression for the energy density, allowance has been made for the fact that the energy is deposited only in the pore space, since the skeleton is taken as nonconducting. We retain the main term in (1.6) for F_0 and use the fact that $j^2 = |(\partial/\partial z)F_0|^2$ to get the distribution of the ohmic loss in the region of the crack vertex:

$$\frac{1}{\sigma_0} j^2\Delta\tau_1 \approx \frac{4a_2^2\Delta\tau_1}{\sigma_0\rho^2(\rho^4 - 2n\rho^2\cos 2\theta + n^2)} \quad (1 \leqslant \rho < \infty). \tag{2.4}$$

We see from (2.4) that the maximum heat production occurs at the vertices of the crack with $\rho = 1$ and $\theta_0 = 0$ and π in a region with characteristic size of the order of β. The heat production at the vertex decreases rapidly away from it in accordance with the law

$$\frac{j^2\Delta\tau_1}{\sigma_0} \sim \frac{\Delta\tau_1}{\sigma_0}\frac{1}{(\beta + \Delta\rho)^2 + (\Delta\theta)^2}, \tag{2.5}$$

where $\Delta\rho = \rho - 1$; $\Delta\theta = (\theta - \theta_0)$. For further estimation, we approximate the distribution of the energy production near the vertex of the crack by means of an expression that takes the following form in a polar coordinate system (r, φ) having its center at the vertex of the crack:

$$E = \frac{a_2^2\Delta\tau_1}{m\sigma_0\beta^2}\exp(-r^2/\beta^2), \tag{2.6}$$

where $r^2 = (x_1 - 1 - n)^2 + y^2$; $y = r\sin\varphi$. It can be shown that for $(r/\beta) \ll 1$ expression (2.6) goes over to (2.5). The model distribution of (2.6) is used to find the solution to (2.1) and (2.2).

The solution to (2.1) can be found by Goodyear's method [6]. We put $U_i = \partial\Psi/\partial x_i$, which shows that equations (2.1) will be satisfied if Ψ satisfies

$$\Delta\Psi = \frac{\alpha_1 M}{\lambda + \alpha_1 M + 2\mu}\zeta(t). \tag{2.7}$$

Knowing Ψ, we can find the stresses in the skeleton, which are defined by the following [5]

$$\sigma_{ij} = 2\mu e_{ij} + \left[\left(\lambda + \alpha_1^2 M\right)e + \alpha_1 M\zeta\right]\delta_{ij}, \tag{2.8}$$

where σ_{ij} is the strain tensor and δ_{ij} is the Kronecker delta. The time dependence in $\zeta(t)$ is described by (2.2), in which we need to incorporate the term Q that allows for the bulk deformation of the liquid on the instantaneous heat production. From (2.3) and (2.6) we get

$$a\Delta\zeta + Q\delta(t) = \partial\zeta/\partial t, \tag{2.9}$$

where $Q = m\beta_1\Delta T\exp(-r^2/\beta^2)$. We put $\zeta = 0$ at the initial instant and use a Hankel transformation [3] with respect to the coordinate r and a Laplace transformation with respect to time to get the solution to (2.9) for an instantaneous energy source:

$$\zeta(t) = m\beta_1\frac{\Delta T}{\eta}\beta^2\exp(-r^2/\eta), \tag{2.10}$$

where $\eta(t) = \beta^2 + 4\alpha t$. Expression (2.10) for $\beta^2\Delta T = \text{const}$ and $\beta \to 0$ becomes the standard expression for an instantaneous point source in an equation of parabolic type.

We substitute (2.10) into (2.7) and use a Hankel transformation to find the ptoential Ψ and correspondingly an expression for $\sigma_{\varphi\varphi}$ from (2.8):

$$\sigma_{\varphi\varphi} = A\Delta\tau_1\frac{1}{r^2}\left[1 - \left(1 + \frac{2r^2}{\eta}\right)\exp(-r^2/\eta)\right], \tag{2.11}$$

where $A = \beta_1\frac{I^2}{(2\pi)^2}\left(\frac{2\beta}{2\beta + \alpha}\right)^2\frac{1}{c_l\,\rho_l\,\sigma_0}\frac{\alpha_1\mu M}{(\lambda + \alpha_1^2 M + 2\mu)}$. As Ψ is independent of φ, it follows from (2.8) that

$\sigma_{rr} = -(2\mu/r)(\partial\Psi/\partial r)$. As would be expected, the stresses decrease with the passage of time when there is instantaneous energy deposition: $\sigma_{\varphi\varphi} \to 0$ for $t \to \infty$ It follows from (2.11) that tensile stresses arise when the liquid is heated, which can disrupt the skeleton if $\sigma_{\varphi\varphi} = \sigma^*$, where σ^* is the limiting tear stress. If on the other hand $(\sigma_{\varphi\varphi} - \sigma_{rr}) = \tau^*$, where τ^* is the limiting shear stress, one can get failure in shear. The maximal values of $\sigma_{\varphi\varphi}$ and $\sigma_{\varphi\varphi} - \sigma_{rr}$ are attained at the initial instant for $r^2/\eta \sim 2$. In this region, the condition for failure in shear with $\alpha \ll 2\beta$ takes the following form for the case of instantaneous energy deposition:

$$\frac{2.4\beta_1}{\pi(\beta l)^2}\frac{1}{\ln\frac{8L}{l}}\left(\frac{E_1}{c_l\rho_l}\right)\frac{\alpha_1\mu M}{\lambda+\alpha_1^2 M+2\mu}\approx\tau^*, \tag{2.12}$$

where $E_1 \approx (I^2\Delta\tau_1/\pi\sigma_0)\ln(8L/l)$ is the energy injected into the medium per unit length of the electrodes. If a current constant over time is passed through the crack, the stresses in the medium can be found by means of a Duhamel integral [3]:

$$\sigma^0_{\varphi\varphi} = \int_0^t \frac{1}{\Delta\tau_1}\sigma_{\varphi\varphi}(t-\tau)\,d\tau.$$

We substitute for $\sigma_{\varphi\varphi}$ from (2.11) into the Duhamel integral to get

$$\sigma^0_{\varphi\varphi} = A\frac{1}{r^2}\left\{t - \frac{c^2}{4a}\left[\left(1+\frac{4at}{c^2}\right)\exp\left(-r^2/c^2+4at\right)-\exp\left(-r^2/c^2\right)\right]\right\}. \tag{2.13}$$

In the case $(r^2/c^2 + 4at) \ll 1$ $(t \gg c^2/4a)$, we have $\sigma^0_{\varphi\varphi} \to A(1/4a)$, so on passing a constant current one gets the maximal tensile stresses at $r = 0$, and their value for $t > c^2/4a$ is independent of time and is governed only by the density of the current emerging from the vertex of the crack. The following relation gives the minimum value of current injected per unit electrode length that will produce shear failure on the assumption that $2\beta \gg \alpha$:

$$I \approx 2\pi\left\{\frac{2}{\beta_1}\tau^* a c_l \rho_l \sigma_0 \frac{\lambda+\alpha_1^2 M+2\mu}{\mu\alpha_1 M}\right\}^{1/2}. \tag{2.14}$$

We note that (2.12) and (2.14) give estimates of the current density required for crack growth in limiting cases. The relation of (2.12) has been derived on the assumption that the liquid does not have time to escape from the energy-production zone during the current pulse. Therefore, infiltration can only increase the I derived from (2.12). The value of I in (2.14) on the other hand has been derived within the framework of the Biot consolidation theory, which assumes that Darcy's linear law applies. It is known [7] that the linear relationship between the infiltration rate and the pressure gradient ceases to apply as the gradient increases. Then the infiltration rate is substantially less than that calculated from Darcy's law. For this reason, (2.14) would give an elevated value of I. Also, the following may have a substantial effect on the dissipation of the pore pressure near the crack vertex: liquid infiltration along the crack, variation in the permeability of the medium because of change in the state of stress and change in the viscosity of the liquid due to the heating.

One can incorporate these factors into (2.14) by using an effective consolidation coefficient a^*. We estimate I for the case $2\beta \gg \alpha$. We put $\tau^* = 2\cdot10^7$ N/m², $\alpha_1\mu M/(\lambda + \alpha_1 M + 2\mu) = 2\cdot10^{10}$ N/m², $a^* = 10^{-1}$ m²/sec, $\sigma_0 = 10^{-3}(\Omega\cdot\text{m})^{-1}$, $\beta_1 = 2\cdot10^{-4}\text{deg}^{-1}$, $c_l = 4.18\cdot10^3$ J/kg·deg, and $\rho_l = 10^3$ kg/m³, and then from (2.14) we get $I = 4\cdot10^2$ A/m. To calculate I from (2.12), one needs to know not only the above parameters but also the value of βl, which determines the characteristic size of the energy deposition region, together with $\Delta\tau_1$, the deposition time, which should satisfy the condition $\Delta\tau_1 \ll (\beta l)^2/a^*$. In principle, $\beta l/4$ is equal to the crack width. However, when the crack is filled with a liquid of high conductivity, it penetrates through the walls, and as a result there is a layer of elevated conductivity around the crack. This may alter the eccentricity of the conducting ellipse and therefore increase the characteristic size of the deposition region. That region can then be characterized by the effective quantity $\beta^* l$. We take the parameters of the medium as being as before, and with $\beta^* l = 5\cdot10^{-3}$ m, $\Delta\tau_1 = 2\cdot10^{-5}$ sec we get from (2.12) that I approximately equals $3\cdot10^2$ A/m, which is less than the value given by (2.14), as would be expected. In accordance with (2.12), I decreases as $\beta^* l$ decreases, $I \sim \beta^* l$. However, there is a lower bound to I, which is associated with the need to obey the condition $2\beta^* \gg \sigma_0/\sigma_1$. If this condition is not obeyed, the current density at the vertex of the crack falls as $\sim 2\beta^*/(2\beta^* + \sigma_0/\sigma_1)$, which reduces the mechanical action of the current and correspondingly makes it necessary to increase the value of I to produce damage. In the case of instantaneous deposition, (2.3) gives the temperature change in the pore space as

$$\Delta T \approx \frac{4I^2}{(\pi\beta^* l)^2}\frac{\Delta\tau_1}{\sigma_0 c_l \rho_l m}.$$

We take the porosity as m = 0.1 to get $\Delta T \approx 83°K$. The corresponding change in the pressure can be estimated if we assume that the compressibility of the skeleton is much less than that of the liquid, the formula being $\Delta p = (b_1/\gamma)\Delta T$, where γ is the bulk compressibility of the liquid. For water, $(\beta_1/\gamma) \approx 4 \cdot 10^5$ N/(m²·deg) [8], and the corresponding pressure change is $\Delta p \approx 3 \cdot 10^7$ N/m², which is comparable with the characteristic damaging stresses. In principle, a phase transition may occur as the temperature increases, which may alter β_1 and γ. However, the equation of state for water [9] implies that β_1 and γ may be taken as constant over fairly wide ranges in temperature and pressure, even when there is a phase transition. These results only represent estimates, because the discussion is based on a model, and also because there is some uncertainty over the choice of β^* and a^*, which should be determined by experiment.

We now discuss the applicability of the model. The crack growth has been considered in the macroscopic approximation, which requires obedience to the condition $\beta^* l \gg d$, where d is the characteristic dimension of the inhomogeneity in the medium, for example the micropore size. Therefore, the estimates apply only for cracks of sufficiently large opening that meet this requirement. To estimate the current density at which crack growth begins, it is necessary to allow for the effects of the highly conducting layer formed around the crack due to the filling with liquid of high conductivity. This liquid infiltrates through the side surface of the crack and may displace the liquid that saturates the porous medium. As a result, a highly conducting region of elliptical form arises around the crack. The solution for the current distribution around this region coincides with that given here. The other formulas defining the current density for the start of damage at the vertex of an elliptical crack of high conductivity also apply. However, in the corresponding formulas one should replace $\beta l/4$, which is equal to the crack width, by $\beta^* l/4$, which characterizes the thickness of the layer around the crack. Also, if the condition $c/l \gg \sigma_0/\sigma_1$ is obeyed and the crack length is much greater than the electrode diameter, the formulas apply even if the condition implying smallness of the electrode diameter relative to the crack width is not obeyed. Also, the Biot theory used does not incorporate inertial effects associated with the establishment of an equilibrium stress distribution. Therefore, the instantaneous deposition model applies only for deposition $\Delta\tau_1 \gg \beta^* l/4c^*$, where c^* is the speed of sound in the skeleton. If on the other hand energy E sufficient to damage the medium is deposited in a time $\Delta\tau_1 < \beta^* l/4c^*$, the damage may be explosive.

Here we have considered the critical current density at which damage occurs around the vertex of the crack. It is then possible for other processes to occur such as further crack growth and the formation of crack rosettes. There may also be a substantial effect on the damage from the hyperfine structure at the ends of the cracks in the saturated porous material. If the initial crack is not elliptical and if the opening at the ends is comparable with the characteristic size of the microcapillaries, then there may be extensive heat production not only near the vertices of the cracks but also around the ends. In that case, the elevated pressure at the ends of the crack may increase the stress-intensity coefficient at the vertex and reduce the current density at which crack growth begins.

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